DOCUMENT RESUME

ED 189 185 TM 800 367

AUTHOR Lacher, S. P.; Torgesen, Ronald M.
TITLE Mathematics in North Dakota. Summary of Performance for Eleventh Grade Mathematics Assessment, Spring, 1978.
INSTITUTION North Dakota State Dept. of Public Instruction, Bismarck.
PUB DATE Nov 78
NOTE 36p.: For a related document, see TM 800 368.

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS *Academic Achievement: *Educational Assessment: Educational Objectives: Grade 11: High Schools: *Quantitative Tests: *Secondary School Mathematics: State Programs: Student Evaluation: Testing Programs
IDENTIFIERS National Assessment of Educational Progress: *North Dakota

ABSTRACT

The purposes of the North Dakota Statewide Assessment of Student Performance in Mathematics are: (1) to meet the public's right to know about the current status of student knowledge throughout the state: (2) to provide information and recommendations for corrective action concerning areas of student performance needing improvement: and (3) to report the results to educational decision-makers. The objectives to be assessed comprise the following cognitive levels: recall and recognition, performing mathematical manipulations, understanding mathematical concepts and processes, problem solving, analyzing problem situations, and appreciation. In addition, data were collected on grade, sex, racial background, home and family background characteristics, general attitudes toward school and mathematics, participation in school programs, mathematics materials used in instruction, and educational and career aspirations. Student performance in basic computation was acceptable but not outstanding. Better attainment was noted on the use of measurement systems, solving consumer problems, algebraic and geometric terminology, reading graphs, applying formulas, and higher order problem solving. Areas of concern include ratio and percents, trigonometric functions, and the interpretation of measures of central tendency. The use of common items facilitated comparison of mathematics performance of North Dakota students with nationwide performance on the National Assessment of Educational Progress. (GSK)

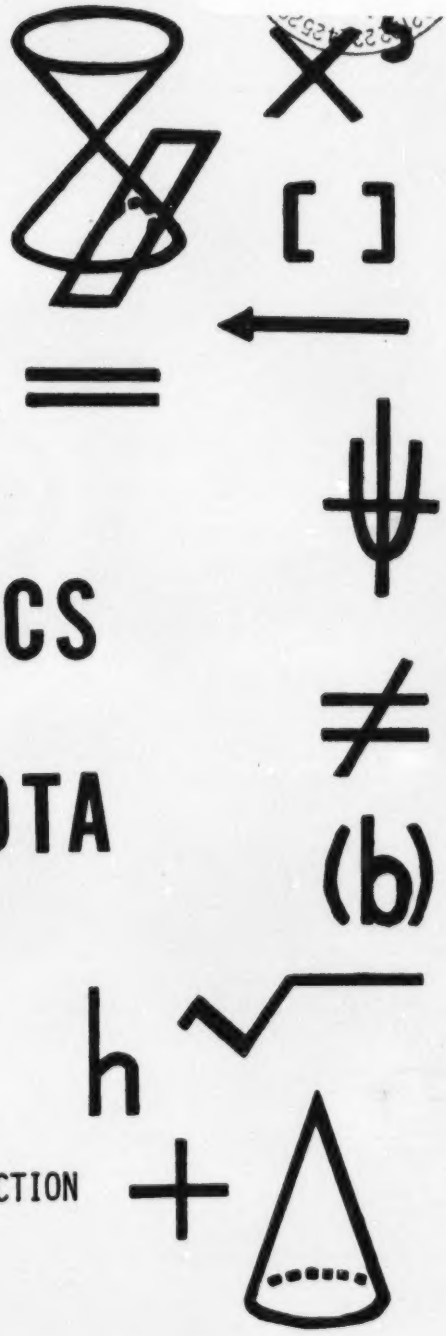
****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED189185

U S DEPARTMENT OF HEALTH,
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

SUMMARY OF PERFORMANCE FOR
ELEVENTH GRADE MATHEMATICS ASSESSMENT
SPRING, 1978



MATHEMATICS IN NORTH DAKOTA

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
R. Torgeson
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

TM 800367

NORTH DAKOTA DEPARTMENT OF PUBLIC INSTRUCTION
H.J. SNORTLAND, SUPERINTENDENT
BISMARCK, NORTH DAKOTA
NOVEMBER, 1978

MATHEMATICS
IN
NORTH DAKOTA

SUMMARY OF PERFORMANCE FOR
ELEVENTH GRADE MATHEMATICS ASSESSMENT
SPRING, 1978

X X X X X X

PREPARED BY
S.R. LACHER, ASSISTANT DIRECTOR
OFFICE OF PLANNING & EVALUATION
DR. RONALD M. TORGESON, DIRECTOR

NORTH DAKOTA DEPARTMENT OF PUBLIC INSTRUCTION
H.J. SNORTLAND, SUPERINTENDENT
BISMARCK, NORTH DAKOTA
NOVEMBER, 1978

TABLE OF CONTENTS

	Page
Acknowledgments	iv
Chapter I Objectives	1
Chapter II Summary of Major Findings	13
Chapter III Performance by Objectives	17
Chapter IV Performance by Clusters	24
Chapter V North Dakota Versus U.S. Performance Comparisons	29

ACKNOWLEDGMENTS

This is the report of the North Dakota Mathematics Assessment for 1977-78. It includes an analysis of the results of the Mathematics tests administered in April, 1978, to eleventh grade students enrolled in randomly selected schools.

The assessment could not have been accomplished without the cooperation of educators throughout the state. Therefore, we wish to extend our sincerest appreciation to the administrators and teachers who coordinated the administration of the tests in the schools and to the following individuals who served on the Advisory Committee.

Mathematics Advisory Committee

Mr. Gary Froelich Bismarck High School	Mrs. Helen Molberg Bottineau High School
Mr. James Lundstrom Mandan High School	Dr. James H. Babb Minot State College
Mr. Harold Mach Minto Public School	Mr. Ken Anderson N.D. State School of Science
Mr. Duane Hanson Kenmare High School	Mr. Donald Sheldon Fargo North High School
Mr. Neil Pederson Carrington High School	Mr. David McCormack North Shore High School
Mr. Keith Petrick Linton High School	Mr. James Johnson Ellendale High School

CHAPTER I

OBJECTIVES

The purposes of the North Dakota Statewide Assessment of Student Performance in Mathematics rest on two important facts: (1) people have a need and a right to know about student performance in our schools and (2) legislators and educators alike have expressed a desire to bring about continuous improvement in instruction.

The purposes of the North Dakota Assessment of Student Performance in Mathematics are:

1. To meet the public's right to know about the current status of student knowledge and skills throughout the state.
2. To provide information about areas of student performance most needing improvement and to make recommendations for corrective action.
3. To report the results to educational decision-makers.

The objectives presented here provide the basics for the selection of exercises to assess the achievement of eleventh grade students in North Dakota. The design chosen for the North Dakota Statewide Mathematics Assessment by the Advisory Committee was that developed for the Minnesota Educational Assessment Program (MEAP) which is similar to the one used by the National Assessment of Educational Progress (NAEP). NAEP is an annual national survey designed to provide information about educational attainments of children and young adults.

The objectives, as developed, comprise the following six cognitive levels:

I. Recall and recognition

II. Performing mathematical manipulations

III. Understanding mathematical concepts and processes

IV. Problem solving

V. Analyzing problem situations

VI. Appreciation

In addition to test items relating to the above objectives, questions requesting information from students in relationship to the following kinds of variables were contained in each exercise package: (1) grade, (2) sex, (3) racial background, (4) home and family background characteristics, (5) general attitudes toward school and mathematics, (6) participation in school programs, (7) mathematics materials used in instruction, and (8) educational and career aspirations.

Ratings of 90 percent, 50 percent and 10 percent values were placed on the objectives to reflect the approximate percent of students across the state who were expected to have been exposed to the content covered by these objectives in the mathematics curricula. These values were viewed as a means for the classification for high, average and low expectancy objectives. The following key to marginal notations contains the same organization and categories as those used by NAEP.

Key to Marginal Notations

90% - General mathematics level

50% - Elementary algebra and geometry through tenth grade level

10% - Advanced math - advanced algebra and analysis

D - De-emphasize

* - Add or emphasize more

SP - Desirable, but not part of math curriculum at present - possibly covered in other areas presently

E - Suggested as enrichment

A total of 47 objectives from the following list of the objectives and marginal notations were used for the assessment. The objectives included in the assessment are denoted with an #.

I. RECALL AND/OR RECOGNITION OF DEFINITIONS, FACTS AND SYMBOLS.

A task assessing this ability will require only that the examinee be able to recognize typical mathematical symbolism, or to recall specific facts. It is the lowest of the levels of cognitive ability in mathematics, but is an essential aspect of achievement. Difficulty level in this category will depend more on exposure to the material and on memory than on developed skill.

The student will demonstrate a competency in the following areas:

A. Knowledge of facts of arithmetic

90% #1. Basic addition, subtraction, multiplication and division facts, using numbers 0 - 9

90% #2. Percent, ratio

90% B. Knowledge of elementary algebra

#1. Recognition of simple algebraic terms, such as: equation, variable, rational and real number, median, mean (average)

C. Knowledge of algebraic symbolsim, such as:

90% 1. $<$, $>$, $=$, $\leq$, $\geq$, $+$, $-$, $\times$, $\div$

50% #2. $A \cup B$, $A \cap B$, $|x|$ (Absolute value)

10% #3. $f(x)$, $\log x$, $\exp x$ (use common exponential notation)
$A \times B$ (Cartesian product)

D. Identification of geometric symbolism, such as:

90% 1. $\parallel$, $\perp$, $\angle PQR$

50% 2. $\cong$, $\sim$

E. Knowledge of terms in algebra, such as:

50% #1. Variable, linear and quadratic equations, coordinates, ordered pair, rational and real numbers

10% #2. Function, inverse

F. In synthetic plan and solid geometry, knowledge of:

50% #1. Terms: parallel, perpendicular, congruent, similar triangles, segment, ray, horizontal, vertical lines

50% #2. Figures: classification of common figures, such as: circles and polygons

G. Knowledge of terms and symbolism in:

50% #1. Elementary analytic geometry - coordinates, ordered pairs, slope, distance, midpoint

10% #2. Trigonometry - right triangle relationships, sine, cosine, tangent

50%,* #3. Elementary probability and statistics - sample space, probability of success or error, mean, median, mode

10% H. Knowledge of a group

50% I. Knowledge of trigonometric functions of special angles

#1. 30^0-60^0-90^0 triangles, 45^0-45^0-90^0 triangles

50% J. Knowledge of basic geometric facts, such as:

#1. Pythagorean theorem, angle relationships in circles (centered and inscribed), mensurational formulas

10% #K. Knowledge of laws of operation for exponents and logarithms

L. Identification of graphs of:

50% 1. Line and circle

10% #2. Parabola, hyperbola, ellipse

M. (Included in "G") Terms of probability

10% N. Knowledge of symbols in logic, such as $\wedge$, $\vee$, ...

O. Knowledge of business and commercial terms in common usage, such as:

90% 1. Gross, net, profit, loss, selling price, cost, discount

SP,* 2. Successive discount, prorate, compoind interest, tax millage, exemption, debt

D,E P. Scientific units (calorie, BTU, wire gauge, etc.)

50% #Q. Knowledge of scientific notation

90%,* R. Knowledge of metric system

#1. Facts of distance, weight and capacity in metric measure

10% #S. Knowledge of necessary and sufficient conditions, converse, inverse, contrapositive and counterexample

90%,D T. Knowledge of English measurement system

90%,* U. Knowledge of computer terminology (e.g., input, processor, memory, output)

II. PERFORM MATHEMATICAL MANIPULATIONS.

The second level of ability will be assessed by means of tasks which require the examinee to carry out single operations and procedures (or sequences of these), that have been previously learned and are specifically requested. Such tasks will require developed skill, but will not require any decision as to which process or sequence of processes is needed (e.g., algorithm). It is in this category that all straightforward computation is included from simple addition to operations with complex numbers; it also includes solution of equations, evaluation of functions, etc. In any case, the tasks which the examinee is required to perform involve only the rote application of learned techniques.

The student will demonstrate a competency in the following:

90% #A. Arithmetic computation; computation with approximate data

#1. Use of four operations in whole, fractional and decimal numbers

#B. Manipulation of algebraic expressions (1st degree polynomials) such as:

50% #1. Combining like terms, removing parentheses, operations with expressions, factoring, evaluating expressions

50% C. Geometric constructions

10% D. Computation with logarithms (practical application)

D E. Slide rule computation

90%,* F. Using a calculator

90% #G. Using conversion relations in measurement (within system being used - not from one system to another)

D H. Using carpenter and engineer rules

10% I. Evaluation of determinants; operations with matrices

J. Solving simultaneous equations

50% #1. Linear

10% 2. Linear and quadratic or quadratic

D K. Nomographs

D,E L. Application of tests of divisibility

90% #M. Finding square roots - by tables, logarithms, calculators, etc.

10% #N. Interpolation and extrapolation with a table

10% O. Finding relative error of a measurement

10% #P. Synthetic division

10% #Q. Expanding a binomial

90% #R. Reading all types of graphs

90% #S. Using formulas (areas of triangles, circles, rectangles, perimeter of polygons, simple interest, etc.)

T. Solving equations and inequalities

50% #1. In one variable (including absolute value)

a, proportions

10% #2. Solve quadratics (i.e., by factoring or quadratic formula)

III. UNDERSTAND MATHEMATICAL CONCEPTS AND PROCESSES.

This third level will include tasks which demonstrate understanding of concepts and of mathematical processes through the ability to transform or translate from one type of "language" or symbolism to another. Since the demonstration of comprehension must involve communication, this level will assess comprehension through tasks which include the following possible kinds of translations or transformations within a mathematical context:

--verbal to mathematical (e.g., words to symbols)
--mathematical to verbal (e.g., symbolism to verbal)
--mathematical to mathematical (e.g., translating from one kind of representation to another like an equation to a graph of the equation)
--mathematical to physical (e.g., use of charts to explain fractions)
--physical to mathematical (e.g., developing formulas for physical phenomena)
--verbal to verbal (e.g., explanation)

The student will demonstrate competency in the following:

A. Make a graph of a function

50% #1. Linear

10% #2. Quadratic and higher degree

#B. Finding the equation of a graph

10% #1. Linear

10% #2. Quadratic and higher

D,E C. Making a paper slide rule

D,E D. Numerals in other bases

10% #E. Demonstrating the properties of a mathematical system such as rotations of a square, or a finite number system

F. (Omit)

D G. Explaining the long division algorithm in terms of successive subtractions

10% H. Identifying a quadratic function and on an exponential function

D,E I. Changing compound interest into series summation

90% #J. Interpreting statistical data, mean, median, mode, etc.

50% #K. Translating a verbal statement into a mathematical sentence, and conversely

50% #L. Illustrating a geometric theorem by making sketches

IV. SOLVING MATHEMATICAL PROBLEMS--SOCIAL, TECHNICAL AND ACADEMIC.

Assessment of the ability to solve problems requires the examinee to demonstrate the ability to select knowledge, skills, information and techniques which are needed to solve a particular problem and to apply such background in actually solving the problem.

Included in an assessment of the ability to solve problems will be tasks ranging from routine to unfamiliar, from specific to abstract, and from those whose solutions are straightforward to those which require ingenuity and insight.

Included under IV will be much of the consumer mathematics used by the majority of adults. Also included will be the ability to follow a proof, find a flaw in a proof, construct a deductive proof, as in a plane geometry problem.

The common characteristic of tasks in this category will be that they require the individual to analyze a problem and determine a sequence of steps which will lead to a clearly specified outcome (whether the outcome is finding the cost of a purchase of providing a theorem).

The student will demonstrate a competency in:

10%	#A.	Solution of triangles using trigonometric ratios (such as using sin, cos and tan)
10%	B.	Proving trigonometric identities
	C.	Solving verbal problems in algebra
50%		#1. Solving simple linear equations in one variable
50%		2. Solving singular variable inequalities
10%		3. Solving second degree equations and inequalities with 2 or more variables
10%	D.	Proving geometric originals
90%,*	#E.	Interpreting tables and graphs
90%,*	#F.	Applying formulas
10%	G.	Evaluation of determinants (same as II "I")
10%	#H.	Computing with complex numbers
D,E	I.	Solve three dimensional locus problems
D,E	J.	Using Maxwell diagrams in solving force problems
D,E	K.	Navigation problems using vectors
50%	#L.	Locating a flaw in geometric proof
50%	#M.	Locating a flaw in algebraic proof
10%	N.	Solving problems involving series and summation
E	O.	Solving problems in symbolic logic
SP	P.	Solving surveying problems
SP	Q.	Measuring, cutting and folding patterns
SP	R.	Solving shop problems
SP	S.	Solving bookkeeping problems

T. Solving time problems

90% #1. Time elapsed problems

90% #2. Time zone problems

90%,* U. Estimation

50%,* #V. Translation of verbal or mathematical problem into flow chart or diagram

90% #W. Solve consumer problems such as: budgets, taxes, insurance, checking accounts, etc.

V. USING MATHEMATICS AND MATHEMATICAL REASONING TO ANALYZE PROBLEM SITUATIONS, DEFINE PROBLEMS, FORMULATE HYPOTHESES, MAKE DECISIONS AND VERIFY RESULTS.

This level is a combination of those high level mathematical abilities which are open-ended and which require the use of mathematical techniques and patterns of thought in an independent and constructive way.

Tasks in this category include those which assess the ability to transfer and utilize knowledge in new situations, to recognize patterns, to draw conclusions from given data, to plan for the future on the basis of present information and to use mathematical reasoning to make optimum decisions.

Tasks in this category also include the ability to recognize the existence of a problem, the ability to state it formally, the ability to formulate hypotheses and the ability to ascertain if the problem has a unique solution. Assessment of the sufficiency of conditions and the determination of the minimum conditions necessary for proof, the disproof of hypotheses by counter-example and proof by induction all come under this heading.

The student will exhibit problem-solving behaviors related to:

50%,* #A. Geometric experiments including drawing figures, making constructions, measurements, models and folding paper in order to discover generalizations

50%,* #B. Recognizing patterns and making generalizations about numerical and algebraic configurations

90% #C. Solving novel problems, puzzles and recreations (from easy to more difficult)

90% #D. Comparative buying

90% E. Planning personal finances

90% F. Budgeting

90% G. Cost estimating, such as planning a trip, remodeling a home

90% #H. Discovering fallacies in consumer advertising involving statistical data and graphs

90% I. Detecting flaws in arguments, such as advertising and propaganda

D,E J. Assembly and presentation of statistical evidence in support of an argument

E K. Curve fitting

90% L. Making a survey: defining problem, sampling, data collection, etc.

50%,* M. Use of computer to solve original problems

90% N. After solving a problem, is interested in whether this solution is correct or not

90%,* O. Given an open-ended problem situation, the student will present more than one possible solution

P. Problem solving techniques:

50%,* #1. Counterexamples

50%,* #2. Use of a simpler case

50%,* 3. Looking at extremes

50%,* #4. Assume you know the answer

50%,* #5. Analysis of problems

(Hints of varying degree of helpfulness will be presented.)

VI. RECOGNIZING THE IMPORTANCE AND RELEVANCE OF MATHEMATICS TO THE INDIVIDUAL AND TO SOCIETY.

From age 9 and up, there should be a recognition of the importance and relevance of mathematics to the individual and to society. This subobjective (VI.A) does not necessarily involve enjoyment of mathematics or participation in the development of ideas, but rather it focuses on the acceptance of mathematics as being worthwhile--i.e., the individual recognizes that mathematics is necessary whether or not he uses it or enjoys studying it. For example, the individual should recognize the contribution that mathematics has made to the progress of civilization, especially in the sciences. There should also be appreciation of the elegance, economy and techniques of mathematics. Of course, the level of sophistication of such appreciations should increase with age; nevertheless, some manifestation of these attitudes should appear at all age levels.

Students should exhibit an appreciation for:

A. The contribution of mathematics to civilization

50% 1. The role of Euclidean geometry

50% 2. Arabic numeration

50% 3. Famous persons and their contributions:

a. Gauss
b. Newton
c. Descartes
d. Galois
e. Einstein
f. Noether
g. Pythagoras

B. The enjoyment of mathematics

In addition to having an appreciation for the importance of mathematics, the individual should also enjoy the subject and its specialized techniques (e.g., using compasses and working with numbers). Emphasis should be placed on the enjoyment involved in acquiring a knowledge of mathematics and in the satisfaction gained from using it rather than on the amount that is learned. The corollary of this is also important--i.e., the individual should not hate or fear mathematics. These attitudinal goals are especially important during the school years since they are likely to influence how much mathematics an individual is willing to study and, therefore, have at his disposal.

We would suggest that some measure of the students' enjoyment of mathematics be included in the assessment exercises or the student questionnaire.

C. Using the content and techniques of mathematics

When the mathematics is relevant and appropriate, individuals should use what they have learned. Because the amount of knowledge varies with age level, evidence of a willingness to use mathematics will take different forms at different age levels.

D. Participation in the learning of mathematics beyond that which is merely required and actively seeking to further personal development in the area of mathematics

The fourth subgoal relates to the individual's development of a curiosity about mathematics as well as a readiness to engage in

activities in this area (i.e., independent of school and/or job assignments). In contrast to the objectives in other categories, independent action rather than reaction is stressed. This goal emphasizes that the individual should actively seek participation and further development of his skills in mathematics (as indicated by such things as reading about the "new" math and tackling strange looking problems). This is opposed to merely passing judgment or using the principles learned when this was required. It is expected that such interests will not develop before the age of 13; however, once developed, they will probably carry through into adult life.

CHAPTER II

SUMMARY OF MAJOR FINDINGS

Mathematics tests were administered to a stratified random sample of students enrolled in grade 11. A total of 1422 students were actually tested out of a population of 12,546 students.

The test instruments were administered during April, 1978, by the staff of each school selected to participate in the assessment study. While an approximate time for test completion was suggested, it was considered liberal enough to designate the instrument as a "power test."

The report which follows contains a summary of student performance by objective and by cluster. The success level indicates the percent of correct responses for each objective. This level is the number of correct responses for each objective divided by the total number of possible correct responses, multiplied by one hundred. Relative to the mastery of objectives, it should be noted that criterion-referenced instruments like those used in this assessment are intended to determine student performance. Conversely, standardized norm-referenced tests are intended to determine the abilities of students by comparing them with other groups of students. Thus, these results cannot be combined into a single performance score as in the case of norm-referenced test results.

While norm-referenced comparisons were not made, the test included items made available by the National Assessment of Educational Progress (NAEP). The inclusion of the NAEP exercises enabled a comparison to be made of North

Dakota student performance with the performance of national and regional groups as measured by the NAEP program.

Open-ended exercises were scored by Department of Public Instruction personnel and Central Data Processing produced the output necessary for data analysis. Planning and Evaluation personnel analyzed the data and prepared the written reports of the results.

Statewide mathematics results were analyzed on the basis of two logical groupings of exercises: objectives and clusters of items. Exercises were selected to measure specific mathematics objectives, but because several objectives and individual exercises can be placed into common categories more consistent with common curricular designations, an additional grouping of exercises into clusters was attempted. Each of these clusters incorporates several objectives and provides a sufficient number of exercises to allow a generalization of results.

Based upon the analysis of mathematics objectives and clusters, the following conclusions can be drawn regarding mathematics performance of North Dakota students in the eleventh grade.

1. When one considers the mathematics programs of North Dakota and the varying amount of exposure to items relating to each objective, it can be said that student performance for objectives indicates a generally high level on a statewide basis. Performance was judged to be acceptable or above on 50 of the 60 objectives.
2. Student performance on items relating to basic computation was acceptable but not outstanding. Better attainment was noted on exercises requiring the use of measurement systems including metrics.
3. Strength was evident on exercises that asked students to solve

consumer problems.

4. Student performance on mathematics clusters was also of a general high level. Strength classification included performance in areas of basic and advanced problem solving, measurement systems including a basic knowledge of the metric system, recognition of geometric figures, and interpreting graphs, tables and maps.
5. Students continue to have problems with the concept of fractions and computations with fractions. These areas of need relate to both common and decimal fractions.
6. Students performed lower than expected in areas relating to the basic knowledge of area, perimeter, volume and other properties of geomentric figures applied to specific situations.
7. As expected, students pursuing a more extensive program of mathematics study performed better on most test items than did those who merely met the minimum requirements for graduation. This was true not only for the more difficult items but for all levels of competencies tested.

Fifty-nine items for seventeen-year-olds were drawn in their exact form from the National Assessment of Educational Progress (NAEP) item bank. Because these items were identical, comparisons can be made between North Dakota and the nation.

An examination of the data indicates the North Dakota students generally perform at a favorable level when compared with other students in the nation at grade eleven.

Mathematics educators at the local level can use these results as a tool for the evaluation of their instructional programs in mathematics and to aid

in setting priorities for instruction. It should be noted that the strengths and weaknesses suggested are based on general observations and may not be appropriate for specific instructional programs. Therefore, school district personnel should consider the suggestions in light of local programs in making judgments regarding the performance levels indicated.

CHAPTER III

PERFORMANCE BY OBJECTIVES

This portion of the report contains a summary of student performance by objective. It should be noted that the performance level will reflect the exposure that students have had to the objectives tested. This exposure will range from basic general math courses to the more advanced courses on the eleventh grade level.

In North Dakota, students are required to obtain one credit in mathematics for graduation from high school. The students in this assessment sample met this requirement in the following manner: 14.3% completed only a general or basic mathematics course, 35% completed only the Algebra I course, 16.3% completed both Algebra I and Geometry, and 33.8% completed at least one course beyond the Algebra I and Geometry level.

The following five basic levels of competencies were tested:

1. RECALLING AND/OR RECOGNITION OF DEFINITIONS, FACTS AND SYMBOLS.
   A task assessing this ability requires that the examinee be able to recognize typical mathematical symbolism, or to recall specific facts.
2. PERFORMING MATHEMATICAL MANIPULATIONS.
   The second level of ability includes tasks which require the examinee to carry out single operations and procedures (or sequences of these), that have been previously learned and are specifically requested.

3. UNDERSTANDING MATHEMATICAL CONCEPTS AND PROCESSES.

   This third level includes tasks which demonstrate understanding of concepts and of mathematical processes through the ability to transform or translate from one type of "language" or symbolism to another.

4. SOLVING MATHEMATICAL PROBLEMS--SOCIAL, TECHNICAL AND ACADEMIC.

   Assessment of the ability to solve problems requires the examinee to demonstrate the ability to select knowledge, skills, information and techniques which are needed to solve a particular problem and to apply such background in actually solving the problem.

5. USING MATHEMATICS AND MATHEMATICAL REASONING TO ANALYZE PROBLEM SITUATIONS, DEFINE PROBLEMS, FORMULATE HYPOTHESES, MAKE DECISIONS AND VERIFY RESULTS.

   This level is a combination of those higher level mathematical abilities which are open-ended and which require the use of mathematical techniques and patterns of thought in an independent and constructive way.

Results by objective are presented in Table 1. When the exposure ratings indicated on the list of objectives are compared with the performance of eleventh grade students reported in Table 1, the analysis can be used to suggest areas of strength and weakness of mathematics performance. Only objectives which were measured by three or more exercises are included.

Generalizable strengths of North Dakota students are related to the following objectives:

IA1 Recall of basic facts of addition, subtraction, multiplication and division

IE1 Knowledge of algebraic terminology

IF1 Knowledge of terms; parallel, similar, ray, etc.

IL2 Identification of graphs

IIG Measurement

IIR Reading graphs

IIS Using formulas

IIT1 Solving equations with one variable

IIIE Properties of a mathematical system

IIIK Translating a verbal statement into a mathematical sentence

IVH Computing with complex numbers

IVW Solve consumer problems

Student performance on items relating to basic computation was acceptable but not outstanding. Better attainment was noted on exercises requiring the use of measurement systems. The results reflect the emphasis placed on metric measurement during the past school year. Strength was also evident on exercises that asked students to solve consumer problems. Additional generalizable strengths relate to knowledge of algebraic and geometric terminology, data interpretation including identification of reading graphs, applying formulas, and higher order problem solving.

The following objectives are suggested as needs at the seventeen-year old level:

IA2 Knowledge of the concepts of percents and ratios

IG3 Knowledge of terms; elementary probability and statistics

II1 Knowledge of trigonometry functions; 30-60-90 triangle; 45-45-90 triangle

IJ1 Knowledge of geometric facts

IIIJ Interpreting statistical data, mean, median, mode

While the objectives suggested, as needs covered a wide range of topics, none concern the basic functions of mathematics, measurement or consumer

mathematics. Areas of concern include ratio and percents, trigonometric functions and the interpretation of measures of central tendency.

A summary of the results by objective is presented in Table 1.

TABLE 1
Summary of Eleventh Grade Student Performance by Objective

	Objective	No. of Items	State Success Level	Number of Students
IA1	Knowledge of basic addition, subtraction, multiplication and division	3	97.2	1,422
IA2	Knowledge of percent and ratio	6	54.5	939
IB1	Knowledge of simple algebraic terms	1	69.3	473
IC2	Knowledge of AUB, A∩B, |x| (absolute value)	1	61.2	466
IC3	Knowledge of f(x), log x, exp x, exponential notation, Cartesian product	4	32.4	1,422
IE1	Knowledge of terms; variables, coordinate, etc.	7	52.4	1,422
IE2	Knowledge of terms; function, inverse	2	35.3	939
IF1	Knowledge of terms; parallel, similar, ray, etc.	9	71.6	1,422
IG1	Knowledge of terms; elementary analytic geometry--slope, etc.	1	44.0	473
IG2	Knowledge of terms; trigonometry--sine, cosine, right triangles, etc.	3	18.8	483
IG3	Knowledge of terms; elementary probability and statistics	2	34.2	939
II1	Knowledge of trigonometry functions; 30-60-90 triangles, 45-45-90 triangles	3	22.6	1,422
IJ1	Knowledge of geometric facts	3	34.4	1,422

Table 1 (Cont.)

	Objective	No. of Items	State Success Level	Number of Students
IK1	Knowledge of laws for exponents and logarithms	2	45.6	473
IL2	Identify graphs; parabola, hyperbola, ellipse	3	35.9	466
IQ1	Knowledge of scientific notation	1	73.0	466
IR1	Knowledge of the metric system	6	79.3	1,422
IS	Knowledge of necessary and sufficient conditions, converse, etc.	1	42.9	483
IIA	Computation with approximate data	4	68.0	956
IIA1	Computation; use of for operations in whole, fractional and decimal numbers	21	76.2	1,422
IIB	Manipulation of algebraic expressions (1st degree polynomials)	4	44.7	1,422
IIB1	Manipulation of algebraic expressions; combining like terms, etc.	4	44.7	1,422
IIG	Conversion relations in measurement (within system being used)	3	54.2	1,422
IIJ1	Solving simultaneous equations (linear)	1	35.9	473
IIM	Finding square root	1	82.4	466
IIN	Interpolation and extrapolation with a table	4	63.8	949
IIP	Synthetic division	1	4.2	473
IIQ	Expanding a binomial	2	27.4	949
IIR	Reading graphs	5	87.7	939
IIS	Using formulas (areas of triangles, circles, perimeter of polygons, etc.)	3	73.0	939

Table 1 (Cont.)

	Objective	No. of Items	State Success Level	Number of Students
IIT1	Solve equations and inequalities in one variable	8	61.5	1,422
IIT2	Solve equations and inequalities of quadratics	4	30.9	949
IIIA1	Make a graph of a function (linear)	2	34.3	939
IIIA2	Make a graph of a function (quadratic and higher degree)	1	15.0	473
IIIB1	Finding equation of graph (linear)	1	11.8	483
IIIB2	Finding equation of graph (quadratic and higher degree)	1	3.0	466
IIIE	Demonstrating the properties of a mathematical system	3	70.7	939
IIIJ	Interpreting statistical data, mean, median, mode	4	16.8	956
IIIK	Translating verbal statement into mathematical sentence	5	54.8	1,422
IIIL	Illustrating geometric theorem by making sketches	1	51.9	466
IVA	Solution of triangles using trigonometric ratios	2	18.6	949
IVC1	Solving verbal problems (simple linear equations in one variable)	1	39.3	473
IVE	Interpreting tables and graphs	2	65.8	473
IVF	Applying formulas	5	57.8	1,422
IVH	Computing with complex numbers	3	35.7	483
IVL	Locating a flaw in geometric proof	1	19.7	473
IVM	Locating a flaw in algebraic proof	1	11.8	466

Table 1 (Cont.)

	Objective	No. of Items	State Success Level	Number of Students
IVT1	Solving time problems; time elapsed	4	75.2	939
IVT2	Solving time zone problems	1	34.6	483
IVU	Estimation	2	61.6	949
IVW	Solve consumer problems; budgets, taxes, insurance, etc.	9	74.4	1,422
VA	Geometric experiments; drawing figures, measurements, models, etc.	2	42.4	939
VB	Recognize patterns and make generalizations about configurations	3	47.7	939
VC	Solving novel problems, puzzles and recreations	1	29.0	483
VD	Comparative buying	3	56.4	1,422
VH	Discover fallacies in consumer ad (statistical data and charts)	1	53.6	466
VP1	Problem solving; counterexamples	1	73.2	473
VP2	Problem solving; use of a similar case	1	13.3	483
VP4	Problem solving; assume you know the answer	1	82.6	466
VP5	Problem solving; analysis of problems	2	92.9	939

C H A P T E R I V

PERFORMANCE BY CLUSTERS

Because several objectives and individual exercises fall into categories considered consistent with common curricular designations, an addtional clustering of exercises was completed. Each of these clusters incorporates several objectives and provides a sufficient number of exercises so that generalizations of the results can be made.

Based on the expected exposure level for the objectives, strengths and weaknesses are suggested. The definitions and general content of each of the eleventh grade clusters are described as follows:

C1. COMPUTATION WITH WHOLE NUMBERS.

Basic facts and addition, subtraction, multiplication, division and taking square roots.

C2. CONCEPTS AND COMPUTATION WITH COMMON FRACTIONS.

The concepts of ratio and equivalent fractions; addition, subtraction, multiplication and division with common fractions.

C3. CONCEPTS AND COMPUTATION WITH DECIMALS.

Addition, subtraction, multiplication and division with decimals; conversion of decimals to percent and common fractions.

P1. PROPERTIES OF NUMBERS.

Deals with odd and even numbers, positive and negative

numbers, rational numbers and real numbers; also deals with factors, exponents and properties of number systems.

G1. RECOGNITION OF GEOMETRIC FIGURES AND RELATIONS BETWEEN FIGURES.

Recognition of angles, polygons, ellipses and parabola; also included are congruence and similarity relations for triangles.

G2. APPLICATIONS OF GEOMETRIC PROPERTIES: Basic knowledge of area, perimeter, volume and other properties of geometric figures applied to specific situations.

A1: ALGEBRAIC EXPRESSIONS.

Involves the knowledge necessary to manipulate algebraic expressions and to solve equations.

A2. ALGEBRAIC APPLICATIONS.

The methods of algebra are used in situations requiring the solutions of algebraic equations.

I2. INTERPRETING GRAPHS, TABLES AND MAPS.

Picture graphs and other representations of data are presented for interpretation and determination of accurate conclusions.

PS1. BASIC PROBLEM SOLVING.

Verbally stated problems that can be translated into equation form and then solved by means of simple arithmetic computations.

PS2. ADVANCED PROBLEM SOLVING.

Recognition and use of the heuristics of problem solving and applying these to problem situations.

M1. MEASUREMENT SYSTEMS.

Use of common units of measurement. The great majority of exercises (six out of seven) deal with a basic knowledge of the metric system.

SP. STATISTICS AND PROBABILITY.

Deals with basic notions of the probability of an event; also deals with the concepts of mean, mode and median.

S1. SETS.

Exercises related to the general concepts of sets and subsets as well as the basic operations on sets.

T1. TRIGONOMETRY.

Deals with basic knowledge and applications of the sine, cosine, tangent and cotangent functions.

C4. CONSUMER PROBLEMS.

Deals with concepts of interest, installment buying, balancing checking accounts and comparative buying.

T2. TIME.

Includes problems relating to time zones, train schedules and elapsed time.

Table 2 includes the student performance for each cluster. The scores indicated are mean scores for all exercises placed within the cluster.

TABLE 2
Classification of Seventeen-Year-Old Performance
For Seventeen Clusters

Cluster	No. of Items	Student Performance Mean %
C1. Computation with whole numbers	13	88.5
C2. Concepts and compuation with common fractions	9	61.2

Table 2 (Cont.)

	Cluster	No. of Items	Student Performance Mean %
C3.	Concepts and computation with decimals	10	62.0
C4.	Consumer problems	11	77.1
P1.	Properties of numbers	12	58.6
G1.	Recognition of geometric figures and relations between figures	17	67.8
G2.	Applications of geometric properties	16	46.2
A1.	Algebraic expressions	11	56.9
A2.	Algebraic applications	17	41.3
I2.	Interpreting graphs, tables and maps	13	72.7
PS1.	Basic problem solving	6	74.1
PS2.	Advanced problem solving	18	43.3
M1.	Measurement systems	9	69.2
SP.	Statistics and probability	6	24.4
S1.	Sets	4	46.1
T1.	Trigonometry	6	19.6
T2.	Time	5	67.1

An analysis of the data presented in Table 2 indicates that performance on 7 of the 17 clusters reflect strengths, while performance on 4 clusters represents areas of need. Performance on the other clusters was considered acceptable. This appears to be consistent with the overall performance on the individual objectives. The highest performance rating was in the area of basic computation (88.5 percent). While this score was considered acceptable, it was felt that there should have been a greater indication of

strength. Other clusters in the strength classification included performance in areas related to basic and advanced problem solving, measurement systems including a basic knowledge of the metric system, recognition of geometric figures and relations between figures and interpreting graphs, tables and maps.

As in a previous assessment relating to sixth grade mathematics performance, students continue to have problems with the concept of fractions and computations using fractions. These problems are related to both common and decimal fractions.

Students also performed at a lower level on the statistics and probability cluster and trigonometry cluster. Exposure to these concepts in the schools included in the sample may be somewhat limited and was reflected in the scores. It is, however, of some concern that students did not score as high as expected in the cluster relating to the basic knowledge of area, perimeter, volume and other properties of geometric figures applied to specific situations.

It was encouraging to note that North Dakota eleventh grade students show a basic knowledge in areas of consumer interests and metric measurement. The performance on metric items reflects the emphases on inservice work during the past school year.

CHAPTER V

NORTH DAKOTA VERSUS U.S. PERFORMANCE COMPARISON

Several of the mathematics items completed by North Dakota students were identical in form to those used by National Assessment of Educational Progress (NAEP). The use of common items facilitates the comparison of the mathematics performance of North Dakota students at grade eleven with the performance of similar students in the nation. The test items are identified according to the objective they measure and their location in the specific test booklet. The comparisons of North Dakota student performance on common objectives and items are presented in Table 3.

TABLE 3
Comparison of Eleventh Grade Student Performance on Common Objectives with National Performance

Objective		Pkg. No.	Item No.	Performance Level In Percents	
				N.D.	NAEP
IA2	Knowledge of percent and ratio	1	3	46.7	51.1
IB1	Knowledge of simple algebraic terms	1	34	69.3	77.8
IC2	Knowledge of AUB, A∩B, |x| (absolute value)	2	34	61.2	57.9
IG3	Knowledge of terms; elementary probability and statistics	1 2	15 41	35.7 32.8	28.6 31.2
IJ1	Knowledge of geometric facts	2	3	55.6	54.9

Table 3 (Cont.)

	Objective	Pkg. No.	Item No.	Performance Level In Percents	
				N.D.	NAEP
IQ1	Knowledge of scientific notation	2	44	73.0	64.1
IS	Knowledge of necessary and sufficient conditions	3	11	42.9	51.1
IIA	Arithmetic computation and approximation	1	6	68.9	73.8
		1	8A	95.6	97.0
		1	8B	93.9	93.2
		1	8C	86.3	89.9
		1	8D	95.3	94.7
		2	9	68.7	68.5
		2	27	45.5	40.3
		2	33	93.1	91.2
		3	4	38.7	40.7
		3	17	83.0	83.2
		3	43	63.6	76.6
		3	45	82.4	67.3
		3	48	50.3	42.6
IIB	Manipulation of algebraic expressions	1	48	31.9	42.9
		3	36	60.9	68.8
IIB1	Manipulation of algebraic expressions; combining like terms	1	4	80.5	79.8
		2	7	17.6	31.1
IIG	Conversion relations in measurement	3	32	89.4	81.0
IIM	Finding square root	2	6	82.4	77.6
IIN	Interpolation and extrapolation with a table	3	37	87.4	39.0
IIR	Reading graphs	1	41	87.7	79.4
IIT1	Solve equations and inequalities in one variable	1	7	80.1	78.8
		1	25	83.1	80.8
		2	38	93.1	95.0
		3	6	71.6	72.9
IIT2	Solve equations and inequalities of quadratics	2	22	70.0	66.4
		3	10	32.5	28.0
IIIA1	Make a graph of a function (linear)	1	38	41.9	48.9

Table 3 (Cont.)

	Objective	Pkg. No.	Item No.	Performance Level In Percents	
				N.D.	NAEP
IIIE	Demonstrating the properties of a mathematical system	1	11	80.3	74.6
		2	12A	61.2	65.3
		2	12B	61.2	67.8
IIIK	Translating verbal statements into mathematical sentences	2	24	57.7	43.0
		3	28	71.2	70.4
IVA	Solution of triangles using trigonometric ratios	3	33	23.8	27.4
IVE	Interpreting tables and graphs	1	18A	53.1	52.2
		1	18B	78.4	71.8
IVF	Applying formulas	1	51	35.5	31.8
		2	50	75.3	70.6
		3	3	25.3	30.3
		3	22	70.0	66.4
IVT1	Solving time problems; time elapsed	1	43	86.9	82.6
		2	42	41.0	43.1
IVT2	Solving time zone problems	3	13	34.6	24.6
IVW	Solve consumer problems; budgets, taxes, insurance etc.	1	30	94.1	86.2
		1	46	75.3	71.0
		3	8	95.7	89.4
		3	39	87.0	83.0
VA	Geometric experiments	2	8	50.6	50.6
VD	Comparative buying	1	27	33.8	35.2
		3	52	51.6	53.0

Statistical tests were not calculated to determine whether North Dakota student performance differs significantly from student performance on the national level. However, when a difference of 5 or more percentage points is used for comparison purposes, an examination of the data for the 59 common items reveals that North Dakota students performed higher than the nation on 15 items, lower than the nation on 9 items, and about the same as the nation on 35 items.